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Kuchta D.,
Ryńca R.

MULTICRITERIA EVALUATION OF UNIVERSITIES

The paper proposes an approach to multicriteria ranking of universities. One of various methods of linguistic multicriteria evaluation was selected and a set of criteria proposed. The chosen method and criteria were applied to rank three universities: one state university and two private ones. The results of the experiment are given. Basic information about linguistic multicriteria evaluation and fuzzy numbers are also presented

1 Fuzzy numbers as models of linguistic terms

In many practical problems, in which non-mathematicians are involved, we face the problem of the need of some quantitative data and the inability of the persons asked to give them. What is worse, very often the data (e.g. scores in questionnaires) are given somehow, because they have to be given, but in fact they do no reflect the true opinions of the persons asked – because these persons find it difficult to express their opinions in numbers, and especially to do this coherently among each other. It is easier for them to use natural language expression. But automatic systems need numerical data for calculation and decision making support. That is why we need a quasi-natural language which will be offered to the persons asked and a translation system of expressing linguistic terms in a mathematical form, possible to be processed by computers.

Fuzzy numbers offer such a translation possibility. Fuzzy numbers [1] can be, to make it as simple as possible, defined as functions, so called membership functions, determined on a set **X,** being a subset of the set of real numbers. A fuzzy number *A* is linked to a membership function $U(x)$ with the following properties and interpretation: $U(x) \in [0,1]$ for all $x \in \mathbf{X}$ and $U(x)$ expresses to which extent the adjective linked to *A*, let us denote it *ADJECTIVE(A)*, is true for *x*. For each *A* there will exist exactly one *ADJECTIVE(A)*.

We will consider the following fuzzy numbers, defined in the domain $[1,9]$:

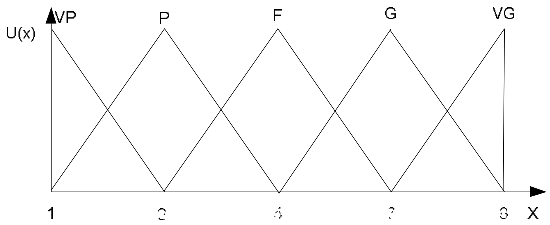

Fig. 1: *Fuzzy numbers used in the paper together with their corresponding adjectives (VP – very poor, P – poor, F – fair, G – good, VG – very good) and their membership functions ([2])*

Thus we will consider five fuzzy numbers: one linked to the adjective “very poor” (VP), with the membership function *U(x)* such that

$$U(x) = \begin{cases} \frac{3 - x}{2} \text{ for } x \in [1,3] \\ 0 \text{ otherwise} \end{cases} . \qquad (1)$$

Formula (1) means that if and only if an object gets, while being evaluated according to a selected criterion, a grade between 1 and 3, it is (according to the selected criterion) very poor to a certain positive extent. For example, if it gets grade 1, it is very poor to the highest possible degree, i.e.1, if it gets grade 2, it is very poor to the degree 0.5, but if it gets a grade from interval $[3,9]$, it is very poor to the zero extent - thus, not at all. The idea is that if the questioned persons do not find it easy to give crisp grades, but prefer to use a natural language, they will be asked to say simply "this is, according to the selected criterion, very poor", and the corresponding algorithm will understand this expression according to (1). Of course, formula (1) is not "taken for heaven", but elaborated by experts in translation between natural language and fuzzy numbers (e.g.[3]) on the basis on a series of experiments. Formula (1) and the corresponding fuzzy number for the adjective "very poor" will be denoted in short as a triple of crisp numbers (1,1,3) – this is the usual notation for so called triangular fuzzy numbers.

In an analogous way we assume that the experts have determined the forms of fuzzy numbers, presented in Fig.1, corresponding to adjectives "poor" (P,(1,3,5)), "fair" (F,(3,5,7)), "good" (G,(5,7,9)) and "very good" (VG,(7,9,9)). It is is easy to notice than an object may be at the same time e.g. poor and fair to some degrees (e.g. if it gets grade 4). That is what the fuzziness consists in: the boundaries between individual notions are not sharp, which is a feature of natural language.

If we have a fuzzy number $(a_1, a_2, a_3)(a_1 \le a_2 \le a_3)$, than its multiplication by a crisp number t >0 gives by definition a fuzzy number (ta_1, ta_2, ta_3). The sum of two fuzzy numbers $(a_1, a_2, a_3)(a_1 \le a_2 \le a_3)$ and $(b_1, b_2, b_3)(b_1 \le b_2 \le b_3)$ gives a fuzzy numbers $(a_1 + b_1, a_2 + b_2, a_3 + b_3)$. The multiplication of fuzzy two numbers is defined analogously ([4]).

A big challenge while dealing with fuzzy numbers is their comparability. Contrary to the crisp numbers, it is not always unequivocal to say which of two fuzzy numbers is to be regarded as bigger. If we have several fuzzy numbers, it may be difficult to rank them. Fig. 1 illustrates the problem: the membership functions presented there overlap each other and this overlapping may be even more "advanced", so that any ranking might be much less unequivocal than it is in Fig. 1. E.g. if we added in Fig. 1 a fuzzy number (1,5,9), it would be not at all obvious how to rank it with respect to the other fuzzy numbers. There are several ranking procedures proposed in the literature (e.g. [2,4,5,6]). The ranking procedure should be adopted to the attitude and opinions of the decision maker. E.g. if the decision maker was a pessimist, the fuzzy number (2,5,7) would be higher in his eyes than the fuzzy number (1,3,9) – because he would rather take into account the pessimistic, lowest possible values where both membership functions are positive - thus the number 2 in case of the fuzzy number (2,5,7) and the number 1 in case of the fuzzy number (1,3,9). If he was an optimist, he would say the fuzzy number (1,3,9) is higher, because of the optimistic numbers 9 and 7. If he was neutral, a kind of average would be a criterion to him, e.g. the numbers 5 and 3, but averages of fuzzy numbers can be calculated in many other ways too ([4]). Here we will have to make a choice of a method of comparing fuzzy numbers and everyone using them to evaluate objects will be faced with this problem.

2 multicriteria crisp evaluation

In many situations there is a necessity to rank a certain group of objects, but the criterion is not unique. This is e.g. the case of universities. In press many rankings of universities appear and many different criteria are selected. In each case when there are several evaluation criteria it is necessary to aggregate all the evaluations into one number, taking into account the weights of individual criteria. What is more, quite often individual criteria have subcriteria, and the subcriteria evaluations have to aggregated to get the main criteria evaluations – it is only then that the final ranking can be determined. There may also be several levels of subriteria.

Abstracting for the moment from fuzzy numbers and linguistic expressions, let us suppose that the evaluations are made in crisp numbers. Let us suppose we have n_0 main criteria $C^0_{j_0}, j_0 = 1,...,n_0$, with weights $w\left(C^0_{j_0}\right), j_0 = 1,...,n_0$ summing up to one. Each main criterion may have $n_1\left(C^0_{j_0}\right)$ subcriteria $C^1_{j_0,j_1}, j_1 = 1,...,n_1\left(C^0_{j_0}\right)$ with weights $w\left(C^1_{j_0,j_1}\right), j_1 = 1,...,n_1\left(C^0_{j_0}\right)$, also summing up to 1. And so forth: each subcriterion $C^1_{j_0,j_1}$ may have $n_2\left(C^1_{j_0,j_1}\right)$ subcriteria $C^2_{j_0,j_1,j_2}, j_2 = 1,...,n_2\left(C^1_{j_0,j_1}\right)$ with weights $w\left(C^2_{j_0,j_1,j_2}\right), j_2 = 1,...,n_2\left(C^1_{j_0,j_1}\right)$ and each subcriterion $C^2_{j_0,j_1,j_2}$ may have further $n_3\left(C^2_{j_0,j_1,j_2}\right)$ subcriteria $C^3_{j_0,j_1,j_2,j_3}, j_3 = 1,...,n_3\left(C^2_{j_0,j_1,j_2}\right)$ with weights $w\left(C^3_{j_0,j_1,j_2,j_3}\right), j_3 = 1,...,n_3\left(C^2_{j_0,j_1,j_2}\right)$. This may of course continue, but in the application proposed in the present paper we will have just four levels of criteria. For all the criteria we will need the evaluations – from various groups of people, averaged somehow. Lets us suppose there are i=1,...,M objects being evaluated. If a criterion $C^0_{j_0}, j_0 = 1,...,n_0$ does have subcriteria, the corresponding evaluation (of the i-th object according to the criteria $C^0_{j_0}$), denoted as $E\left(i, C^0_{j_0}\right)$, are calculated from the following formula:

$$E\left(i, C^0_{j_0}\right) = \sum_{j_1=1}^{n_1\left(C^1_{j_0,j_1}\right)} w\left(C^1_{j_0,j_1}\right) E\left(i, C^1_{j_0,j_1}\right) \qquad (2)$$

If a criterion $C^0_{j_0}, j_0 = 1,...,n_0$ does not have subcriteria, the evaluations are give directly by the persons being questioned (we do not discuss here the problem of aggregating the evaluations given by various persons to one single evaluation, usually it will a simple average). Formula (2) is generalized to the other levels, to $E\left(i, C^1_{j_0,j_1}\right), j_1 = 1,...,n_1\left(C^0_{j_0}\right)$, $E\left(i, C^2_{j_0,j_1,j_2}\right), j_2 = 1,...,n_2\left(C^1_{j_0,j_1}\right)$, $E\left(i, C^3_{j_0,j_1,j_2,j_3}\right), j_3 = 1,...,n_3\left(C^2_{j_0,j_1,j_2}\right)$, i=1,...,M: if a criterion has subcriteria, the evaluation of the individual objects according to this criterion is calculated by a formula analogous to (2), if a criterion does not have subcriteria, the respective evaluations are given directly by the persons evaluating the objects.

Finally, we are in position to determine the final ranking of the M objects, it is given by the ranks

$$R(i) = \sum_{j_0=1}^{n_0} w\left(C^0_{j_0}\right) E\left(i, C^0_{j_0}\right) \qquad (3)$$

The higher the rank, the better.

3 multicriteria fuzzy (linguistic) evaluation

In case the persons evaluating various objects are not willing or able to give coherent crisp evaluations of the objects according to different criteria, they may be allowed to use linguistic expressions. Often those presented in Fig. 1 are used. These persons are not worried about the mathematical translation of their evaluation, they just use the expressions “poor”, “fair” etc. But for the system these expressions correspond to membership functions, to fuzzy numbers. The fuzzy evaluations according to subcriteria are multiplied by weights. The weights may be fuzzy or crisp. In case they are fuzzy, a "language" to describe them me be chosen, similar to that for criteria (Fig. 1), composed of expressions like "high", "low", etc. In case they are crisp, they should be chosen in such a way that on each subcriteria level they add up to 1. The weighted evaluations are added up criteria level by criteria level and finally we get for each object *i*=1,...,*M* n_0 weighted fuzzy evaluations $E\left(i, w\left(C_{j_0}^0\right) \cdot C_{j_0}^0\right)$ with membership functions $U\left(i, C_{j_0}^0\right)$, using fuzzy equivalents of formulae like (2).

As mentioned in Section 2, a ranking based on fuzzy numbers is usually not unequivocal. Here we adopt the ranking procedure used in [2]. Its general idea is as follows: For each criteria the following two crisp numbers are calculated:

$$MAX(j_0) = \sup_x \left\{ \exists i = 1,\dots,M : U\left(i, C_{j_0}^0\right)(x) > 0 \right\} \tag{4}$$

$$MIN(j_0) = \inf_x \left\{ \exists i = 1,\dots,M : U\left(i, C_{j_0}^0\right)(x) > 0 \right\} \tag{5}$$

$MAX(j_0)$ represents, for the criterion j_0, the highest crisp evaluation that was given for this criterion with a positive value of one of the membership functions from Fig.1 - thus in a sense the ideal object according to this criterion. $MIN(j_0)$ represents the contrary: the lowest evaluation given with a positive value of one of the membership functions from Fig.1, thus the worst object according to the considered criterion. Then for each object *i* a (crisp) distance between its evaluation according to each criterion main $E\left(i, C_{j_0}^0\right)$ and the values calculated in (5) and (6) is determined (details can be found in [2]): $DIST[i, MAX(j_0)]$ and $DIST[i, MIN(j_0)]$: the first one represents the distance of the object from the "positive ideal" and the second one the distance of the object from the "negative ideal" . The final ranking of the objects is calculated on the basis of the sums $\sum_{j_0=1}^{n_0} DIST[i, MAX(j_0)]$ and $\sum_{j_0=1}^{n_0} DIST[i, MIN(j_0)]$, where the first one should be as small as possible and the second one as big as possible. An optimism level of the decision maker, $\alpha \in [0,1]$, is selected. The greater α, the greater is the weight of the criterion $\sum_{j_0=1}^{n_0} DIST[i, MAX(j_0)] \to \min$, the smaller α, the greater is the weight of the criterion $\sum_{j_0=1}^{n_0} DIST[i, MIN(j_0)] \to \max$. It is so, because an optimist sets optimistic goals and wants to be as close as possible to the best solution, and a pessimist only wishes to be as far as possible from the worst solution. Details again can be found in [2].

4criteria for university evaluation

We propose to use the criteria for university evaluation listed below. These criteria have been chosen on the basis of a pilot questionnaire performed in two Polish universities. We have $n_0 = 3$ main criteria (student satisfaction, university teacher satisfaction, university management satisfaction), which are composed of several subcriteria. Each criterion should be given a weight.

– C_1^0 : student satisfaction

- $C_{1,1}^{1}$: teaching process
- $C_{1,1,1}^{2}$: teaching staff
- $C_{1,1,2}^{2}$: teaching methods
- $C_{1,1,3}^{2}$: teaching infrastructure
- $C_{1,1,4}^{2}$: organisational aspect of the teaching process
- $C_{1,2}^{1}$: administration functioning
- $C_{1,2,1}^{2}$: dean office
- $C_{1,2,2}^{2}$: recruitment process
- $C_{1,2,3}^{2}$: financing system
- $C_{1,3}^{1}$: university infrastructure
- $C_{1,3,1}^{2}$: university library
- $C_{1,3,2}^{2}$: free computer access
- $C_{1,3,3}^{2}$: campus
- $C_{1,3,4}^{2}$: possibility of developing own interests
- $C_{1,4}^{1}$:university prestige
- $C_{1,5}^{1}$: professional perspectives
- C_{2}^{0} : teacher satisfaction
- $C_{2,1}^{1}$: remuneration policy
- $C_{2,1,1}^{2}$: wages height
- $C_{2,1,2}^{2}$: bonuses
- $C_{2,1,3}^{2}$: social and fringe benefits
- $C_{2,2}^{1}$: working conditions
- $C_{2,2,1}^{2}$: safety at the work place
- $C_{2,2,2}^{2}$: work organisation
- $C_{2,2,2,1}^{3}$: holiday length
- $C_{2,2,2,2}^{3}$: information access
- $C_{2,2,2,3}^{3}$: working hours
- $C_{2,2,3}^{2}$: possibility of professional development
- $C_{2,2,3,1}^{3}$: possibility of conference participation
- $C_{2,2,3,2}^{3}$: possibility of acquiring scientific degrees

- $C^3_{2,2,3,3}$: number and quality of scientific seminars
- $C^2_{2,2,4}$: atmosphere at the work place
- $C^1_{2,3}$: university infrastructure
- $C^2_{2,3,1}$: equipment of the offices
- $C^2_{2,3,2}$: equipment of the laboratories
- $C^2_{2,3,3}$: university building standards
- $C^1_{2,4}$: university prestige
- C^0_{3} : university management satisfaction
- $C^1_{3,1}$: university scientific influence
- $C^2_{3,1,1}$: right to confer scientific titles
- $C^2_{3,1,2}$: number of scientific titles conferred
- $C^2_{3,1,3}$: staff potential
- $C^3_{3,1,3,1}$: reliability
- $C^3_{3,1,3,2}$: ethical attitude
- $C^3_{3,1,3,3}$: expert knowledge
- $C^3_{3,1,3,4}$: languages knowledge
- $C^3_{3,1,3,5}$: own development, continuous learning
- $C^3_{3,1,3,6}$: ability to use technical equipment
- $C^3_{3,1,3,7}$: ability to work and solve problems by themselves
- $C^3_{3,1,3,8}$: ability to generate initiatives
- $C^3_{3,1,3,9}$: creativity
- $C^2_{3,1,4}$: number of citations
- $C^2_{3,1,5}$: number of PhD students
- $C^2_{3,1,6}$: number of publications
- $C^2_{3,1,7}$: number of accreditations passed successfully
- $C^1_{3,2}$: university development
- $C^2_{3,2,1}$: university infrastructure
- $C^2_{3,2,2}$: number of faculties
- $C^2_{3,2,3}$: internationalisation of the studies
- $C^3_{3,2,3,1}$: number of programmes taught entirely in a foreign language

- $C^{3}_{3,2,3,2}$: number of students studying in a foreign language
- $C^{3}_{3,2,3,3}$: international exchange of students
- $C^{3}_{3,2,3,4}$: multi-culturality of the students
- $C^{3}_{3,2,3,5}$: number of foreign teachers
- $C^{3}_{3,2,3,6}$: number of open lectures in foreign languages
- $C^{3}_{3,2,3,7}$: number of summer schools
- $C^{2}_{3,2,4}$: number of students
- $C^{2}_{3,2,5}$: number of branches in other towns
- $C^{1}_{3,3}$: economic effectiveness
- $C^{2}_{3,3,1}$: cost level
- $C^{2}_{3,3,2}$: use of European funds
- $C^{2}_{3,3,3}$: use of industrial funds
- $C^{1}_{3,4}$: university prestige
- $C^{2}_{3,4,1}$: cooperation with the industry
- $C^{3}_{3,4,1,1}$: number of common projects
- $C^{3}_{3,4,1,2}$: number of orders from the industry
- $C^{3}_{3,4,1,3}$: will of the industry to employ the university graduates
- $C^{2}_{3,4,2}$: cooperation with other universities
- $C^{3}_{3,4,2,1}$: students exchange
- $C^{3}_{3,4,2,2}$: number of common projects

In the questionnaire each criterion should be accompanied by an explanation what exactly the person asked should understand under it. As we can see, the main criteria divide the persons to be asked into three groups: students, university teachers and members of the university management. Thus, the universities are evaluated from three different perspectives.

5 real world universities comparison - results

We used the criteria presented in section 4 in a real word experiment. We asked a selected group of students, teachers and managers of three Polish universities (one state university (U1) and two private university: U2 and U3) to evaluate their university according to the criteria proposed above, using the language from Fig.1 (thus only the expressions: very poor, poor, fair, good, very good – the answers were averaged according to one of the methods described in [4]). For simplicity reasons we assumed that the criteria weights are crisp and equal on each criteria level. We chose three optimism levels: 0 (the decision maker is a complete pessimist), 0,5 (the decision maker is neither a pessimist nor an optimist) and 1 (the decision maker is a complete optimist).

The persons participating in the questioning were asked to give evaluations only on the lowest criteria levels, which were then aggregated to the higher levels and finally to the final ranking of the three universities.

The following evaluations were given:

***Tab. 1:** Evaluations given by the students representatives of three universities (main criterion C_1^0):*

University	*U1*	*U2*	*U3*
$C_{1,1,1}^2$	Good	Fair	Fair
$C_{1,1,2}^2$	Fair	Good	Poor
$C_{1,1,3}^2$	Fair	Poor	Poor
$C_{1,1,4}^2$	Fair	Good	Fair
$C_{1,2,1}^2$	Poor	Poor	Poor
$C_{1,2,2}^2$	Fair	Fair	Fair
$C_{1,2,3}^2$	Poor	Poor	Poor
$C_{1,3,1}^2$	Fair	Fair	Fair
$C_{1,3,2}^2$	Fair	Fair	Fair
$C_{1,3,3}^2$	Fair	Fair	Fair
$C_{1,3,4}^2$	Fair	Poor	Poor
$C_{1,4}^1$	Good	Fair	Fair
$C_{1,5}^1$	Good	Fair	Fair

***Tab. 2:** Evaluations given by the teaching staff representatives of three universities (main criterion C_2^0):*

University	*U1*	*U2*	*U3*
$C_{2,1,1}^2$	Fair	Fair	Fair
$C_{2,1,2}^2$	Fair	Fair	Poor
$C_{2,1,3}^2$	Good	Fair	Fair
$C_{2,2,1}^2$	Fair	Fair	Fair
$C_{2,2,2,1}^3$	Good	Fair	Fair
$C_{2,2,2,2}^3$	Fair	Fair	Fair
$C_{2,2,2,3}^3$	Fair	Fair	Fair
$C_{2,2,3,1}^3$	Fair	Fair	Poor
$C_{2,2,3,2}^3$	Good	Fair	Fair
$C_{2,2,3,3}^3$	Fair	Fair	Fair
$C_{2,2,4}^2$	Fair	Fair	Fair
$C_{2,3,1}^2$	Fair	Poor	Poor
$C_{2,3,2}^2$	Fair	Fair	Fair
$C_{2,3,3}^2$	Fair	Poor	Fair
$C_{2,4}^1$	Fair	Fair	Fair

Tab. 3: *Evaluations given by the management representatives of three universities (main criterion C_3^0):*

University	U1	U2	U3
$C_{3,1,1}^2$	Good	Fair	Fair
$C_{3,1,2}^2$	Good	Fair	Fair
$C_{3,1,3,1}^3$	Good	Good	Good
$C_{3,1,3,2}^3$	Very good	Very good	Good
$C_{3,1,3,3}^3$	Very good	Very good	Very good
$C_{3,1,3,4}^3$	Good	Fair	Fair
$C_{3,1,3,5}^3$	Good	Good	Fair
$C_{3,1,3,6}^3$	Fair	Fair	Fair
$C_{3,1,3,7}^3$	Good	Good	Good
$C_{3,1,3,8}^3$	Fair	Fair	Fair
$C_{3,1,3,9}^3$	Good	Good	Good
$C_{3,1,4}^2$	Fair	Poor	Poor
$C_{3,1,5}^2$	Good	Very poor	Very poor
$C_{3,1,6}^2$	Good	Fair	Fair
$C_{3,1,7}^2$	Fair	Poor	Poor
$C_{3,2,1}^2$	Fair	Fair	Fair
$C_{3,2,2}^2$	Fair	Fair	Poor
$C_{3,2,3,1}^3$	Fair	Poor	Poor
$C_{3,2,3,2}^3$	Fair	Poor	Poor
$C_{3,2,3,3}^3$	Fair	Poor	Poor
$C_{3,2,3,4}^3$	Fair	Fair	Fair
$C_{3,2,3,5}^3$	Good	Fair	Fair
$C_{3,2,3,6}^3$	Good	Fair	Fair
$C_{3,2,3,7}^3$	Fair	Poor	Poor
$C_{3,2,4}^2$	Fair	Good	Good
$C_{3,2,5}^2$	Fair	Very good	Fair
$C_{3,3,1}^2$	Fair	Fair	Fair
$C_{3,3,2}^2$	Good	Poor	Fair
$C_{3,3,3}^2$	Poor	Poor	Fair
$C_{3,4,1,1}^3$	Good	Poor	Fair
$C_{3,4,1,2}^3$	Fair	Poor	Fair
$C_{3,4,1,3}^3$	Good	Fair	Fair
$C_{3,4,2,1}^3$	Good	Fair	Fair
$C_{3,4,2,2}^3$	Good	Fair	Fair

Independently of the optimism level chosen, we got each time the same overall ranking, which corresponded to the numbering given to the universities before the experiment: the state university turned out to be the best one, the second was university U2 and the third – university U3. It is easy to notice that private universities U2 and U3 are according to some criteria better than the state university U1. However, in the overall ranking the state university turned out to be better. Of course, other criteria weights might change the situation. However, in the experiment performed we did not ask the participants to give criteria weights and assumed them to be equal on each criteria level, because the questionnaire presented to them was already quite long and demanded plenty of time and attention. In the future a reduction of the number of the criteria (the subcriteria levels) might be taken into consideration, in order to reduce the effort linked to the questionnaires.

6 conclusions

The paper contains a proposal of how to rank universities according to multiple criteria. The proposition comprises first of all the criteria themselves and secondly a choice of a ranking method, in which the user can use linguistic expressions, which are then automatically “translated” into quantitative expressions and aggregated into a ranking of universities. The possibility to use a quasi human language assured on one hand a certain ease for the participants of the research, representing various backgrounds and mathematical preparation levels, and on the other hand a coherence in their answers.

Multicriteria ranking can always be questioned, because it has the drawback of aggregating different points of views and different perspectives into one number, and this aggregation depends heavily in the criteria and methods chosen. However, it seems to be difficult to avoid ranking of universities. It is performed by many institutions, published in the press and discussed by the public. It is thus desirable to understand the mechanisms of such rankings and to be able to influence them, so that they reflect the real quality of universities and really help various customers of universities as well as the management of the latter to make right decisions.

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