

RESEARCH ON THE OPTIMAL STRUCTURE FOR FINANCIAL INSTITUTION

The main issue of this article is to find the optimal structure for the financial institution, e.g. to find the optimal portfolio which would give the highest profit at a certain size of the risk. The main subject is a structure of the financial institution, for example, bank and its affiliates. The main problem is to find optimal structure, which could give more benefit if assets of the bank would be allocated according to the risk. The object of this research is to find optimal structure for the bank and its affiliates.

This article will begin with the case where investment assets come in a few discrete types and compare the performance of a unitary structure and a bipartite structure consisting of a subsidiary that holds low-risk assets and does not default and a subsidiary that holds high-risk assets and defaults part of the time.

1. Introduction

The main issue of this article is to find the optimal structure for the financial institution, e.g. to find the optimal portfolio which would give the highest profit at a certain size of the risk. The main subject is a structure of the financial institution, for example, bank and its affiliates. The main problem is to find optimal structure, which could give more benefit if assets of the bank would be allocated according to the risk. The object of this research is to find optimal structure for the bank and its affiliates. The tasks of this article are:

- to find the basic assumptions which creates the conditions under which unitary structure gives effective investments;
- to find the basic assumptions which creates the conditions under which bipartite structure gives effective investments;
- to find the when unitary structure works better than bipartite structure;
- to measure and to compare the profit of these structures.

This article will begin with the case where investment assets come in a few discrete types and compare the performance of a unitary structure and a bipartite structure consisting of a subsidiary that holds low-risk assets and does not default and a subsidiary that holds high-risk assets and defaults part of the time. It becomes obvious that, if high-risk assets are relatively plentiful, the bipartite structure supports

efficient investment more often than the unitary structure does. When high-risk assets are less plentiful, matters are more mixed: although in some cases the bipartite structure continues to be more likely to support efficient investment, there are also cases where a unitary structure is better because the bipartite structure's risky subsidiary distorts investment incentives.

In this article, it is mentioned that a unitary structure works best if the institution's investment opportunities have relatively homogeneous risk levels, efficient (positive-NPV) investments are relatively profitable, and efficient high-risk investments have relatively limited downside. To see why, consider the institution's incentive to substitute inefficient (negative-NPV) high-risk investments for efficient low-risk investments. The benefit from such risk-shifting is the ability to pocket the upside on inefficient high-risk investments while leaving debt holders with much of the downside; the cost is the lost profits on the low-risk investments. If the risk of investments does not vary much, the potential gain in upside is low; if efficient low-risk investments are profitable, the cost in lost profits is high; if efficient high-risk investments have limited downside, the aggregate downside of the institution's intended asset mix is less, and so a risk-shifting institution must absorb more of the downside from the new investments before debt holders are affected. When investment risk is more heterogeneous or efficient investments are less profitable, two subsidiaries may be able to achieve efficient incentives where a unitary structure cannot. Intuitively, dividing assets between a relatively safe and a relatively risky subsidiary may lower the safer subsidiary's debt financing rate and certainly insulates its assets from the downside of riskier investments.

2. Basic assumptions

It is known that a financial institution has access to various investments ("assets"), each of which requires one unit now and returns an amount next period that depends on the asset's type and the state of the world. There are two possible states of the world next period, 1 and 2; for simplicity, both are equally likely. Thus an asset's type can be represented by (e_1, e_2) , where $e_i \geq 0$ is its (gross) return in state i . Assets (especially those that offer an expected value in excess of investors' required returns) are available in limited supply. This reflects the notion that institutions do add some value, i.e., they can find some positive-NPV investments, but these are limited in number.

Accordingly, it is assumed that investment assets fall into two risk classes, low and high. Within each class, there are two sub-types: those that have expected returns less than investors' required return r ("inefficient" or "negative-NPV" assets), and those that have expected returns greater than r ("efficient" or positive-NPV" assets). More precisely, low-risk assets return s in both states, where $s = s_b < r$ for inefficient low-risk assets and $s = s_g > r$ for efficient low-risk assets. High-risk assets return

$(1+\alpha)t$ in state 1 and $(1-\alpha)t$ in state 2, where $\alpha \in (0, 1]$, $t = t_b < r$ for inefficient high-risk assets, and $t = t_g > r$ for efficient high-risk assets. We also assume that $(1-\alpha) t_g < r$, so that efficient high-risk assets do have some downside risk.

As we know, the institution has access to a limited number of investment assets. Let S^* denote the mass of efficient low-risk assets that the institution has access to, and S the mass of inefficient low-risk assets it has access to; similarly, let T^* and T denote the mass of efficient and inefficient high-risk assets it has access to, respectively. We will also assume that the inefficient assets of a given risk class weakly outnumber the efficient assets in that class, i.e., $S \leq S^*$ and $T \leq T^*$; this simplifies analysis without affecting the substance of the results.

3. Optimal unitary structure with discrete asset type

This will be started with the case where investment assets come in a few discrete types and compare the performance of a unitary structure and a bipartite structure consisting of a subsidiary that holds low-risk assets and does not default and a subsidiary that holds high-risk assets and defaults part of the time. It will be shown that, if high-risk assets are relatively plentiful, the bipartite structure supports efficient investment more often than the unitary structure does. When high-risk assets are less plentiful, matters are more mixed: although in some cases the bipartite structure continues to be more likely to support efficient investment, there are also cases where a unitary structure is better because the bipartite structure's risky subsidiary distorts investment incentives. As mentioned above, if efficient asset returns always exceed r , a bipartite structure is never better than a unitary structure.

The first-best investment rule is to fund all assets with expected return greater than r , in which case the institution's size is $I^* \equiv S^* + T^*$. Because of the incentive problem already noted, however, this first-best rule may not be feasible. We first consider the case where the institution tries to fund the efficient portfolio in a unitary (single subsidiary) structure. If investors believe that the institution will choose the efficient portfolio, then they expect that gross portfolio returns (before debt payments) are $S^*s_g + T^*(1+\alpha)t_g$ in state 1 and $S^*s_g + T^*(1-\alpha)t_g$ in state 2, and so they require that the institution promise to pay a face value of per unit of debt. The equality below displays these conditions.

$$R_D = \max \left\{ r, 2r \frac{S^* s_g + T^* (1 - \alpha) t_g}{I^*} \right\}$$

Then the profit in states would be calculated as shown in the Figure 1.

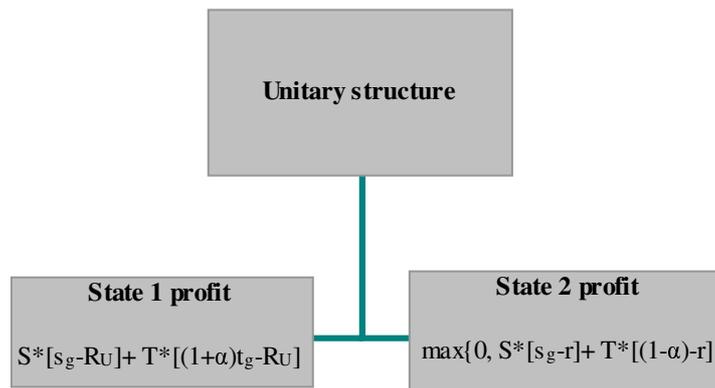


Figure 1. The profit of the unitary structure in state 1 and state 2.

Efficiency of the unitary structure is available if:

➤ The unitary structure cannot support the efficient portfolio if the efficient portfolio is risky (that is, if the efficient portfolio's average state 2 return is less than the required return r).

➤ The unitary structure supports the efficient portfolio if and only if

$$\min\{T, S^*\}[(1+\alpha)t_b - s_g] < S^*(s_g - r) + T^*[(1-\alpha)t_g - r]. \quad (3)$$

➤ The unitary structure is more likely to support the efficient portfolio as the risk of high-risk assets (α) decreases, as the number of efficient low-risk assets (S^*) rises relative to the number of efficient high-risk assets (T^*), as the expected returns of efficient assets (s_g and t_g) increase, or as the expected return of inefficient high-risk assets (t_b) decreases.

It is noted that the right-hand side of 3 is the state 2 payoff to the institution from the efficient portfolio, while the left-hand side is the increase in upside (state 1 return) from replacing $\min\{T, S^*\}$ efficient low-risk (s_g) assets with inefficient high-risk (t_b) assets. Plunging is attractive when this increase in upside offsets the loss of any state 2 income that would accrue under efficient investment. The second term on the right-hand side is negative; lumping high-risk t_g assets together with low-risk s_g assets weakens the institution's resistance to risk-shifting. As the returns on efficient assets (s_g and t_g) increase, the temptation to plunge decreases. Increasing the numbers or riskiness of high-risk assets reduces the state 2 payoff and increases the upside from risk-shifting increases this temptation. When the unitary portfolio defaults in state 2 (the right-hand side of (3) is negative), the unitary structure *always* succumbs to plunging.

The unitary structure is inefficient when the efficient portfolio's aggregate return in state 2 is too small – i.e., when the downside of high-risk assets is too great relative to the returns from low-risk assets. One might then conjecture that separating these two groups into two subsidiaries might improve matters by isolating safer assets from the

“contagion” of riskier assets. As we now show, this conjecture generally holds; the subsidiary with the safer assets is more immune to risk-shifting than a unitary structure.

4. Optimal bipartite structure with discrete asset type

Suppose the institution sets up two subs: one (“Sub A”) is of size S^* and is supposed to hold only the s_g assets; the other (“Sub B”) is of size T^* and is supposed to hold only the t_g assets. If this arrangement is incentive compatible, Sub A pays r on its debt and never defaults, whereas Sub B pays $R_B = 2r - (1 - \alpha)t_g$ on its debt and defaults in state 2. Figure 2 displays these results.

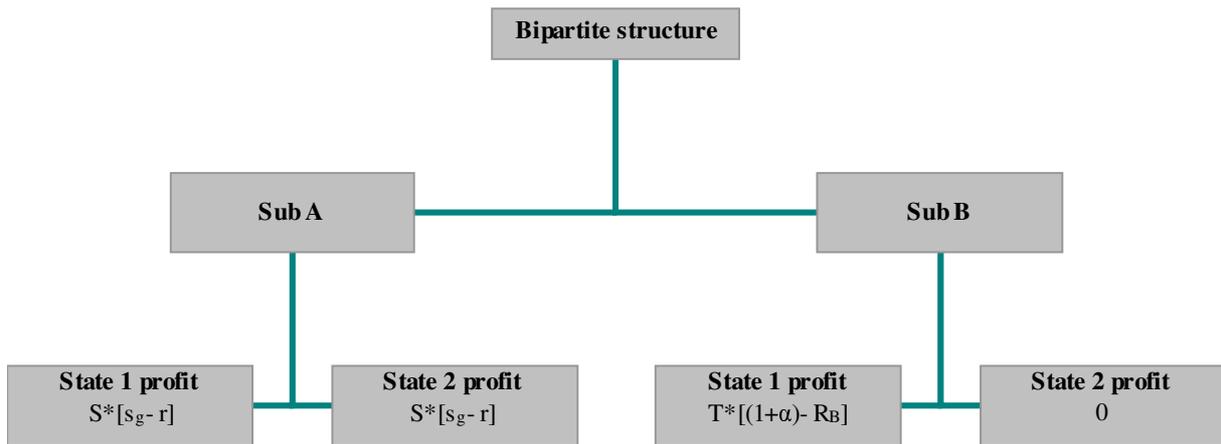


Figure 2. The profit of the bipartite structure in state 1 and state 2.

The question of whether this bipartite structure supports efficient investment is more complex than in the unitary case, since the institution has additional options for asset-substitution: in addition to switching assets in and out of its overall holdings, it can place them in one subsidiary or another, or switch assets between subsidiaries. The next proposition establishes necessary and sufficient conditions for efficiency. Although the general statement is somewhat cumbersome, these conditions simplify considerably in a number of cases, as we will demonstrate shortly.

Efficiency of the bipartite structure is achievable if there are some conditions. The bipartite structure just described supports efficient investment if and only if the following conditions hold:

$$\min\{\underline{I}, S^*\} [(1 + \alpha)t_h - s_z] < S^*(s_z - r), \tag{4}$$

$$t_z - s_z < \frac{1}{2}(1 + \alpha)(t_z - t_h), \tag{5}$$

and either

$$\min\{S^* - T^*, \underline{I}\} [(1 + \alpha)t_h - s_z] < S^*(s_z - r) + T^* \min\{s_z - R_B, R_B - s_z\}, \tag{6}$$

If $S^* > T^*$ (efficient low-risk assets outnumber efficient high-risk assets), or

$$S^* [(1 + \alpha)t_z - s_z] < S^*(s_z - r) + 2T^*(t_z - r), \tag{7}$$

If $S^* \leq T^*$ and $T^*(t_g - r) < S^*(1 + \alpha)(t_g - t_b)$ (replacing S^* of Sub B's t_g assets with t_b assets makes Sub B default all the time); otherwise, only conditions (4) and (5) are required.

Condition 4 precludes plunging. This is weaker than the corresponding condition 3 for the unitary structure. Intuitively, under the target investment mix, efficient low-risk assets are “insulated” from the downside of high-risk assets, making it more costly to get risk-shifting gains in Sub A. This can be seen in Figure 1: under efficient investment, Sub A has higher net returns in state 2 than does the unitary structure, and so the institution has more to lose by plunging in Sub A than by plunging in a unitary structure. Through Sub B, the status quo already has the institution “shifting” the downside on efficient risky assets (t_g) to debt holders, but Sub B's debt is priced accordingly. Effectively, plunging in the unitary structure lets the institution extract value from all debt holders, whereas plunging in the bipartite structure only extracts value from the debt holders of Sub A and is more costly (since the forgone net returns in state 2 are higher).

Condition 6 largely focuses on another type of asset-substitution that we call “flipping,” in which the “safe subsidiary” (Sub A) is filled with high-risk assets while the “risky subsidiary” (Sub B) is filled with low-risk assets. More specifically, a “flip” begins by swapping assets between Sub A and Sub B, after which any low-risk t_g assets remaining in Sub A are replaced by inefficient high-risk t_b assets. When $S^* \leq T^*$, this is dominated by the asset-rotation strategy already described, but when the inequality is reversed flipping may be better. Intuitively, when S^* is small, the institution can plunge in *both* subsidiaries via asset-rotation, but when S^* exceeds T this is impossible. In the second case it may be better to focus all high-risk assets on Sub A so as to maximize net state 1 returns; Sub B is then filled as needed with low-risk assets. When S^* is between T^* and T , it can be shown that condition 6 only binds when $(1 + \alpha)t_g$ is less than R_B , in which case it is the analog of (7), ruling out asset rotations or flips that leave Sub B defaulting all the time.

Intuitively, we know from the discussion of condition 4 above that the bipartite structure is more proof against “plunging” than the unitary structure is. When T exceeds S^* , there are enough bad high-risk assets to completely replace all low-risk ones, so both bipartite and unitary structures allow a total focus on high-risk assets, and the “no plunging” advantage of the bipartite structure is most telling. More formally, we know that the “no plunging” condition 4 for a bipartite structure is weaker than the similar condition 3 for a unitary structure. When T exceeds S^* , condition 6 or 7 (as appropriate) is also weaker than condition 3, and condition 4 implies that the “no rotation” condition 5 holds.

There are two sets of circumstances in which a bipartite structure opens the door to exploitative behavior that would *not* arise under a unitary structure. The first case occurs when efficient low-risk assets outnumber *all* high-risk assets (efficient *and* inefficient) and the low-risk assets are fairly profitable. Here, a bipartite structure may open the door to flipping, whereas the unitary structure may be efficient. Of course, if the number or net return of low-risk assets increases sufficiently, exploiting debt holders never pays, and either structure supports efficient investment.

The other case where the unitary structure dominates occurs when efficient low-risk assets outnumber inefficient high-risk assets and condition 5 is violated. In this case, even if the unitary structure is efficient, the bipartite structure succumbs to asset rotation, taking efficient high-risk assets into Sub A and replacing them in Sub B with inefficient high-risk assets. Our next result gives more details.

There are presented the conditions under which unitary structure supports effective investments while bipartite structure doesn't:

➤ then the bipartite structure does not support efficient investment. Condition 5 is less likely to hold as high-risk asset returns t_g and t_b increase, as low-risk asset returns s_g decrease, and as the risk of high-risk assets (α) decreases;

➤ the unitary structure supports efficient investment if condition 3 holds. Condition 3 requires that

$$T^* < \frac{s_g - r}{s_g - (1 - \alpha)t_b} \cdot S^* < S^* . \quad (8)$$

Condition 8 is more likely to hold as the number S^* and return s_g of efficient low-risk assets increase, as the number of efficient high-risk assets (T^*) decreases, as the risk of high-risk assets (α) decreases, and as investors' required return r decreases.

In second part, condition 8 implies that rotating T^* of t_g assets into Sub A and T^* of t_b assets into Sub B does not increase Sub A's chance of default. If this were not true, then wholesale plunging in the bipartite structure would be attractive, but this would imply that plunging in the unitary structure would also be attractive, contradicting condition 3. If asset rotation is not to make Sub A default, then there cannot be too many efficient high-risk assets vis-a-vis efficient low-risk assets and low-risk assets' returns cannot be too low.

5. Conclusions

This analysis suggests that a unitary structure works best when:

- the spread in risk between different asset types (here α) is relatively small;
- high-risk assets are relatively few in number;
- average expected return of efficient assets (s_g or t_g) is high relative to the required return r , or the average expected return of inefficient assets (t_b) is low.

When these conditions do not hold, a bipartite structure may do better: the safe subsidiary (Sub A) is better-protected from risk-shifting than is the unitary structure, because its assets are insulated from the downside of efficient but high-risk assets; also, the debt of the risky subsidiary (Sub B) is already priced to reflect default risk from efficient high-risk assets. This is always true when inefficient high-risk assets are plentiful.

There are some features of the unitary structure, which were discovered from the analysis, mentioned below:

➤ the unitary structure cannot support the efficient portfolio if the efficient portfolio is risky (that is, if the efficient portfolio's average state 2 return is less than the required return r);

➤ the unitary structure supports the efficient portfolio if and only if the state 2 payoff to the institution from the efficient portfolio is bigger than the increase in upside (state 1 return) from replacing efficient low-risk (s_g) asset with inefficient high-risk (t_b) assets;

➤ the unitary structure is more likely to support the efficient portfolio as the risk of high-risk assets (α) decreases, as the number of efficient low-risk assets (S^*) rises relative to the number of efficient high-risk assets (T^*), as the expected returns of efficient assets (s_g and t_g) increase, or as the expected return of inefficient high-risk assets (t_b) decreases.

The bipartite structure has some features similarly:

➤ the bipartite structure supports the efficient portfolio if the state 1 payoff to the institution from the efficient low risk portfolio is bigger than the increase in upside (state 1 return) from replacing efficient low-risk (s_g) asset with inefficient high-risk (t_b) assets;

➤ the bipartite structure supports the efficient portfolio if inefficient high-risk assets outnumber efficient low-risk assets;

➤ “safe subsidiary” has to be filled with high-risk assets while the “risky subsidiary” has to be filled with low-risk assets.

But bipartite structure may create problems. If efficient low-risk assets are sufficiently profitable and numerous, the bipartite structure may succumb to flipping even when a unitary structure supports efficient investment. If efficient low-risk assets are only more numerous than inefficient high-risk assets, the bipartite structure may still be undermined by asset rotation if high-risk assets are sufficiently profitable relative to low-risk assets.

The critical weakness of a bipartite structure is that the risky subsidiary already defaults with some probability; this reduces the opportunity cost of taking on inefficient high-risk assets into this subsidiary, because some of this cost is borne by debt holders in states of default.

When there are more risk classes of assets, this weakness means that the more risky subsidiary often engages in *some* inefficient risk-shifting. Nevertheless, by

limiting risk-shifting to the risky subsidiary, a bipartite structure may still be able to dominate a unitary structure when the latter succumbs to wholesale plunging.

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